



Contents lists available at ScienceDirect

Chinese Journal of Aeronautics

journal homepage: www.elsevier.com/locate/cja

Approximate Nonlinear Modeling of Aircraft Engine Surge Margin Based on Equilibrium Manifold Expansion

LIU Xiaofeng*, ZHAO Lei

School of Transportation Science and Engineering, Beihang University, Beijing 100191, China

Received 21 July 2011; revised 4 February 2012; accepted 21 March 2012

Abstract

Stable operation of aircraft engine compressions is constrained by rotating surge. In this paper, an approximate nonlinear surge margin model of aircraft engine compression system by using equilibrium manifold is presented. Firstly, this paper gives an overview of the current state of modeling aerodynamic flow instabilities in engine compressors. Secondly, the expansion form of equilibrium manifold is introduced, and the choosing scheduling variable method is discussed. Then, this paper also gives the identification procedure of modeling the approximate nonlinear model. Finally, the modeling and simulations with high pressure (HP) compressor surge margin of the aircraft engine show that this real-time model has the same accuracy with the thermodynamic model, but has simpler structure and shorter computation time.

Keywords: aircraft engine; surge margin; system identification; equilibrium manifold; scheduling variable; perpendicular expansion

1. Introduction

Air and gas flows in aircraft engines are susceptible to instabilities. These flow instabilities are characterized by rapid, unsteady fluctuations in velocity, temperature and pressure. The unsteady fluctuations, associated with compression system surge and stall, are often deterministic and repeatable. They disrupt surrounding flow fields and cause severe stresses on engine components; they also degrade engine efficiency and even cause flow reversals or flame-out [1].

The rapidly increasing requirements for the aircraft engine performance make the aircraft engine components to be exposed to increasing high thermal and aerodynamic loads, and the components are operating

more closely to their physical limits, so compression system surge has become important design and operability issues.

To address these issues, over the years, many research activities by the authors have been done on the modeling of surge for design controllers of aircraft engine compression system. An essential step in controller design is the understanding of the physical phenomena in the system and the development of a mathematical model that describes the most important phenomena. In the literature, various models can be found that describe rotating stall or surge.

For dynamic analysis and controller design in axial compressors, the low-dimensional Greitzer model [2], Moore model [3], and Moore-Greitzer model [4] can be used. The Greitzer and Moore-Greitzer models are capable of predicting transients subsequent to the onset of compressor instability. The Moore-Greitzer model gives rise to three ordinary differential equations model describing the airflow through the compression system. The first is for the amplitude of mass flow rate fluctuations, the second is for the non-dimensional,

*Corresponding author. Tel.: +86-10-82316627.

E-mail address: liuxf@buaa.edu.cn

Foundation items: National Natural Science Foundation of China (61104146); Innovation Plan of Aero Engine Complex System Safety by the Ministry of Education Chang Jiang Scholars of China (IRT0905)

circumferentially averaged mass flow rate through the compressor, and the third is for the non-dimensional total-to-static pressure rise across the compression system. Many authors have extended and modified the model^[5-13]. Inclusion of higher order harmonics was studied by Mansoux, et al.^[14], and other types of compressor characteristics were used by Wang, et al.^[15]. A linear model based on Moore-Greitzer model was developed to predict the dynamic behavior of a compression system subject to a perturbation from steady operating conditions in Ref. [16]. Badmus, et al.^[17] discussed the stall and surge model which was made via the reference of the compression system bifurcation diagrams. The specific bifurcation diagrams are generated using the Moore-Greitzer model. The model described in Ref. [18] starts from a general description by application of principles of conservation of mass, energy, and momentum for a calorically perfect gas. The model can be applied to generic compression systems and can describe the dynamic behavior of axial as well as radial compressors during surge.

Besides the theoretical work on the compression system surge, a lot of experimental work and computational fluid dynamics (CFD) modeling had also been done^[19-22]. Wu and Li^[23] developed a method of effectively selecting the support vectors applied to the asymmetric support vector machines-based surge map modeling framework. The modeling results correctly predict all gathered surge conditions with much less support vectors and kernel of lower orders. A three-dimensional Navier-Stokes solver for modeling unsteady compressor flows has been developed in Ref. [24]. Yoon, et al.^[25] offered experimental test data to validate the proposed mathematical model of the compression system.

Despite of much progress, significant research remains to be done in the direction of modeling the compressor surge margin for control. This paper presents an approach to derive a real-time aircraft engine surge margin model based on nonlinear system's equilibrium manifold expansion form. The effectiveness of the modeling method is evaluated with one twin-spool with a low bypass ratio turbofan engine. The results show that the surge margin model based on equilibrium manifold expansion form can not only satisfy the accuracy of modeling requirement, but also be in compliance with the aircraft physical characteristics.

The outline of this paper is as follows. First, the calculation method for compressor surge margin of aircraft engine is described in Section 2. Then, Section 3 introduces the equilibrium manifold expansion form for modeling the nonlinear system, and discusses the choosing method for scheduling variable. Section 3 also illustrates the identification procedure. Section 4 shows the aircraft engine high pressure (HP) com-

pressor surge margin modeled based on this method, and simulation results. Finally, the conclusions can be obtained in Section 5.

2. Calculation Method for Surge Margin

The ability of a compression system to deliver the required pressure ratio over a wide range of throttle settings is essentially governed by the performance map of the system. As the mass flow rate through the compression system decreases, the pressure rise across the system increases as long as the spool speed of the system is held constant, as shown in Fig. 1. In Fig. 1, π_c is the compressor pressure ratio, $m_{a,cor}$ the corrected air mass flow.

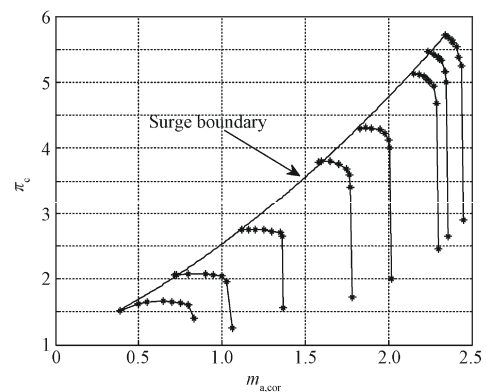


Fig. 1 Compressor performance map.

The provision of sufficient surge margin is an important consideration to aircraft engine safety in any compressor design^[26]. For ensuring the surge boundary, the casual method is based on tests. In this paper, the surge boundary is determined by curve fitting the compressor characteristic's limit values. Firstly, using the compressor working data, the compressor characteristic lines can be plotted at different corrected spool lines, and the surge boundary is lined by selecting the limit values on different corrected spool lines. Hence, the limit values are the intersection points between the corrected spool lines and compressor surge boundary. Interpolation method can be used in order to improving the data fitting accuracy of the surge boundary. It is known by compressor's principle that there is a theoretical start working point, at which the pressure ratio is 1.0 and the air flow is 0, that is, there is no pressurized effect. Through the use of these data, the surge boundary is shown in Fig. 1.

The surge margin can be defined as shown in Fig. 2, and the surge margin can be described as Eq. (1). In Fig. 2, π_s is the pressure ratio at surge, π_w the pressure ratio of working point, n_{max} the maximum spool speed, n_{idle} the idle condition spool speed.

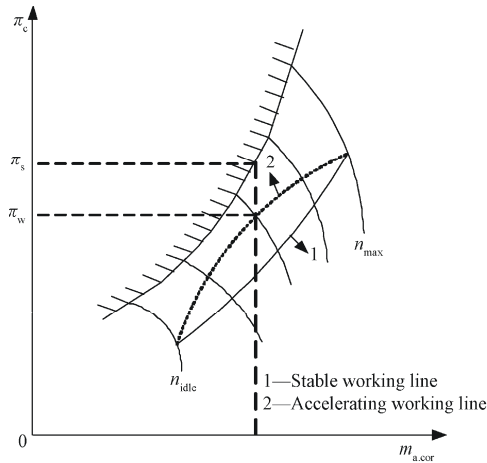


Fig. 2 Definition of surge margin.

$$SM = \left(\frac{\frac{\pi_s}{m_{a,scor}}}{\frac{\pi_w}{m_{a,wcor}}} - 1 \right) \times 100\% \quad (1)$$

where $m_{a,scor}$ is the corrected air mass flow at surge, $m_{a,wcor}$ the corrected air mass flow at the working point.

3. Expansion Model Based on Equilibrium Manifold for Nonlinear System

3.1. Equilibrium manifold description

In this section, the nonlinear system's equilibrium point and equilibrium manifold are introduced firstly.

The nonlinear system is described by the form

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, u) \\ \mathbf{y} = \mathbf{g}(\mathbf{x}, u) \end{cases} \quad (2)$$

where $\mathbf{x} = [x_1 \ x_2 \ \cdots \ x_n]^T$ is the state, $\mathbf{f} = [f_1 \ f_2 \ \cdots \ f_n]^T$ the corresponding function, $\mathbf{y} = [y_1 \ y_2 \ \cdots \ y_r]^T$ the output, $\mathbf{g} = [g_1 \ g_2 \ \cdots \ g_r]^T$ the corresponding function, and u the system input.

1) Equilibrium point of nonlinear system

The point \mathbf{x}_e is an equilibrium point of Eq. (2), if there exists u_e such that

$$\mathbf{f}(\mathbf{x}_e, u_e) = \mathbf{0} \quad (3)$$

Note that, the subscript “e” stands for steady states in this paper.

2) Equilibrium manifold of nonlinear system

The equilibrium family of nonlinear system is a set of equilibrium point, such that

$$(\mathbf{x}_e, \mathbf{y}_e, u_e) \left\{ \begin{array}{l} \mathbf{f}(\mathbf{x}_e, u_e) = \mathbf{0} \\ \mathbf{g}(\mathbf{x}_e, u_e) = \mathbf{y}_e \end{array} \right. \quad (4)$$

According to implicit function theorem^[27], it can be proved that: for the nonlinear system described by Eq. (2), there exists a continuous mapping between these parameters' steady relationships. Generally speaking, the equilibrium points of real nonlinear systems are continuous, and the set of these points can be expressed by using polynomials. The set of equilibrium points is equilibrium manifold^[28].

In this paper, a mapping is introduced to the nonlinear model

$$\sigma = p(\mathbf{x}, u) \quad (5)$$

where σ is called scheduling variable and the constraint for the mapping is that the steady state Eq. (6) must be satisfied.

$$\sigma = p(\mathbf{x}(\sigma), u(\sigma)) \quad (6)$$

So, the equilibrium manifold of the nonlinear system can be written by

$$\begin{cases} \mathbf{x}_e = \mathbf{x}_e(\sigma) \\ \mathbf{y}_e = \mathbf{y}_e(\sigma) \\ u_e = u_e(\sigma) \\ \sigma = p(\mathbf{x}_e(\sigma), u_e(\sigma)) \end{cases} \quad (7)$$

3.2. Equilibrium expansion model for nonlinear system

For each σ , Taylor expansion form of the nonlinear system on the equilibrium point (\mathbf{x}_e, u_e) is written by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \begin{bmatrix} x_{1\delta} \\ x_{2\delta} \\ \vdots \\ x_{n\delta} \end{bmatrix} + \begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \\ \vdots \\ \frac{\partial f_n}{\partial u} \end{bmatrix} u_\delta \quad (8)$$

where

$$\begin{cases} x_{k\delta} = x_k - x_{ke}(\sigma) \\ u_\delta = u - u_e(\sigma) \end{cases} \quad (k=1, 2, \cdots, n) \quad (9)$$

And $\partial f_i / \partial x_j$, $\partial f_i / \partial u$ ($i, j=1, 2, \cdots, n$) is the partial-derivative term of Taylor expansion on the equilibrium points $[x_{1e}(\sigma) \ x_{2e}(\sigma) \ \cdots \ x_{ne}(\sigma) \ u_{1e}(\sigma) \ u_{2e}(\sigma) \ \cdots \ u_{me}(\sigma)]$ determined by σ .

3.3. Method of selecting scheduling variable for expansion model

In the actual calculation, the selection of the appropriate scheduling variables has a huge impact on the model accuracy of the nonlinear system. So the most important step of the modeling is to select the scheduling variables. The following discussion shows the different accuracy of different scheduling variables.

For an n -order single input single output (SISO) nonlinear system, its equilibrium manifold is an $n+1$ -dimensional space curve. The normal-plane function on the equilibrium manifold decided by σ is shown in Eq. (10).

$$\begin{aligned} & \frac{dx_{1e}(\sigma)}{d\sigma}(x_1 - x_{1e}(\sigma)) + \frac{dx_{2e}(\sigma)}{d\sigma}(2x_2 - \\ & x_{2e}(\sigma)) + \dots + \frac{dx_{ne}(\sigma)}{d\sigma}(x_n - x_{ne}(\sigma)) + \\ & \frac{du_e(\sigma)}{d\sigma}(u - u_e(\sigma)) = 0 \end{aligned} \quad (10)$$

For any space point (x, u) , we put it to Eq. (10), and we can get the mapping, $\sigma = p(x, u)$, which corresponds to the shortest distance between the current point with the point on the equilibrium manifold, as shown in Fig. 3.

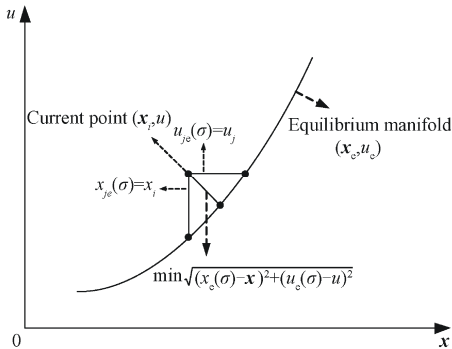


Fig. 3 Sketch map of perpendicular expansion model based on equilibrium manifold.

The scheduling variable selecting method under equilibrium manifold perpendicular expansion form is different from the selecting $\sigma = x_{ke}$. The distance is shorter than the method of selecting $\sigma = x_{ke}$, and the error of the equilibrium manifold expansion model is smaller.

For example, consider a simple nonlinear system:

$$\begin{cases} \dot{x}_1 = -4x_1^2 + x_2^2 + u^2 \\ \dot{x}_2 = x_1^2 + u^2 \end{cases} \quad (11)$$

The equilibrium manifold of this nonlinear system is formulated by Eq. (12).

$$\begin{cases} -4x_{1e}^2 + x_{2e}^2 + u_e^2 = 0 \\ -x_{1e}^2 + u_e^2 = 0 \end{cases} \quad (12)$$

The equilibrium manifold of this nonlinear system is

shown in Fig. 4.

The Taylor expansion model on the equilibrium points (x_{1e}, x_{2e}, u_e) is shown by Eq. (13).

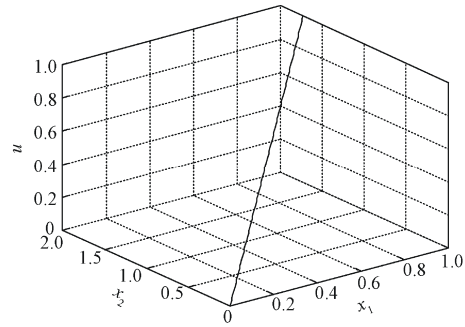


Fig. 4 Equilibrium manifold of the nonlinear system.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -8x_{1e} & 2x_{2e} \\ -2x_{1e} & 0 \end{bmatrix} \begin{bmatrix} x_1 - x_{1e} \\ x_2 - x_{2e} \end{bmatrix} + \begin{bmatrix} 2u_e \\ 2u_e \end{bmatrix} (u - u_e) \quad (13)$$

Firstly, we choose $\sigma = x_{1e}$,

$$\begin{cases} x_{1e} = \sigma \\ x_{2e} = \sqrt{3}\sigma \\ u_e = \sigma \end{cases} \quad (14)$$

The corresponding Jacobian linearization matrix is

$$\begin{cases} A|_{x_{1e}=\sigma} = \begin{bmatrix} -8\sigma & 2\sqrt{3}\sigma \\ -2\sigma & 0 \end{bmatrix} \\ B|_{x_{1e}=\sigma} = \begin{bmatrix} 2\sigma \\ 2\sigma \end{bmatrix} \end{cases} \quad (15)$$

So, the expansion model is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -8x_1 & 2\sqrt{3}x_1 \\ -2x_1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ x_2 - \sqrt{3}x_1 \end{bmatrix} + \begin{bmatrix} 2x_1 \\ 2x_1 \end{bmatrix} (u - x_1) \quad (16)$$

Secondly, we choose $\sigma = x_{2e}$,

$$\begin{cases} x_{1e} = \frac{\sigma}{\sqrt{3}} \\ x_{2e} = \sigma \\ u_e = \frac{\sigma}{\sqrt{3}} \end{cases} \quad (17)$$

The corresponding Jacobian linearization matrix is

$$\begin{cases} A|_{x_{1e}=\sigma} = \begin{bmatrix} -\frac{8\sigma}{\sqrt{3}} & 2\sigma \\ -\frac{2\sigma}{\sqrt{3}} & 0 \end{bmatrix} \\ B|_{x_{1e}=\sigma} = \begin{bmatrix} \frac{2\sigma}{\sqrt{3}} \\ 2\sigma \end{bmatrix} \end{cases} \quad (18)$$

So, the expansion model is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{8x_2}{\sqrt{3}} & 2x_2 \\ -\frac{2x_2}{\sqrt{3}} & 0 \end{bmatrix} \begin{bmatrix} x_1 - \frac{x_2}{\sqrt{3}} \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{2x_2}{\sqrt{3}} \\ \frac{2x_2}{\sqrt{3}} \end{bmatrix} \left(u - \frac{x_2}{\sqrt{3}}\right) \quad (19)$$

Then, the normal-plane function of this equilibrium manifold is formulated by Eq. (20) if we choose $\sigma = x_{1e}$.

$$(x_1 - \sigma) + \sqrt{3}(x_2 - \sqrt{3}\sigma) + (u - \sigma) = 0 \quad (20)$$

And, the mapping function, $\sigma = p(x, u)$, is

$$\sigma = \frac{x_1 + \sqrt{3}x_2 + u}{5} \quad (21)$$

Finally, substituting Eq. (14) and Eq. (21) into Eq. (13), we can obtain perpendicular expansion model of this nonlinear system. Note that, throughout the calculation, whenever we select $\sigma = x_{1e}$ or $\sigma = x_{2e}$, the perpendicular expansion model is the same.

According to three expansion models, we built the Simulink model respectively, and use a step signal as simulation input. Figures 5-6 show the system's two states. From the simulation, we can see that the perpendicular expansion is close to the original system.

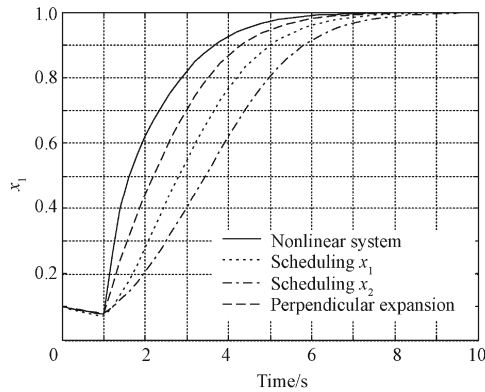


Fig. 5 Comparison of simulations with different expansion methods (x_1).

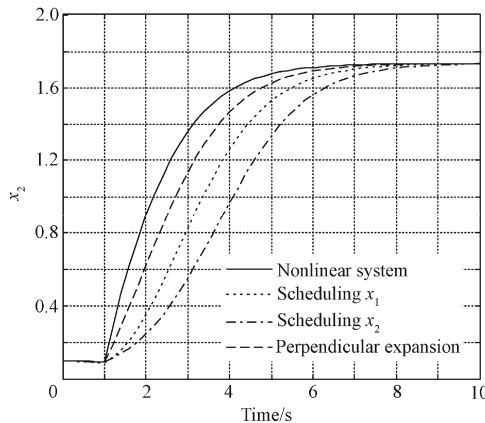


Fig. 6 Comparison of simulations with different expansion methods (x_2).

3.4. Identification of equilibrium manifold expansion model under two steps

As shown in Eqs. (8)-(9), the parameterized form of the equilibrium manifold expansion model is constructed by two items, one is the steady term, and the other is Jacobian matrix.

1) The first step: determine the nonlinear system equilibrium manifold. Suppose that, an SISO system's equilibrium manifold can be presented by a one-dimensional scheduling variable, σ , and the scheduling variable can be a function of the state, input, or any exogenous signal. An SISO equilibrium manifold can be written by

$$\begin{cases} x_{je} = \sum_{i=0}^{n_m} k_{xji} \sigma^i \\ u_e = \sum_{i=0}^{n_m} k_{ui} \sigma^i \end{cases} \quad (j=1, 2, \dots, n) \quad (22)$$

where k represents the corresponding polynomial coefficient, and n_m the order of the equilibrium manifold determined by σ .

2) The second step: derive the partial-derivative terms. The partial-derivative term of Taylor expansion can be written as

$$\begin{cases} \frac{\partial f_m}{\partial x_k} = \sum_{i=0}^{n_d} a_{mki} \sigma^i \\ \frac{\partial f_m}{\partial u} = \sum_{i=0}^{n_d} b_{mi} \sigma^i \end{cases} \quad (m, k=1, 2, \dots, n) \quad (23)$$

where a, b represent the corresponding coefficients, and the dynamic parameters are n_d -order polynomials. Substituting Eqs. (8)-(9) into Eq. (24), we can get Eq. (25).

$$\begin{aligned} \dot{x}_m = & \sum_{l=1}^n \sum_{i=0}^{n_d} a_{mli} \sigma^i (x_l - \sum_{j=0}^{n_m} k_{xlj} \sigma^j) + \\ & \sum_{i=0}^{n_d} b_{mi} \sigma^i (u - \sum_{j=0}^{n_m} k_{uj} \sigma^j) + \frac{\partial^2 f_m}{2\partial l^2} \delta^2 \end{aligned} \quad (24)$$

$$\dot{x}_m = \sum_{l=1}^n \sum_{i=0}^{n_d} a_{mli} \sigma^i \Delta x_l + \sum_{i=0}^{n_d} b_{mi} \sigma^i \Delta u + \frac{\partial^2 f_m}{2\partial l^2} \delta^2 \quad (25)$$

When Eq. (25) is divided by δ^2 , we can get

$$\frac{\dot{x}_m}{\delta^2} = \sum_{l=1}^n \sum_{i=0}^{n_d} a_{mli} \frac{\sigma^i \Delta x_l}{\delta^2} + \sum_{i=0}^{n_d} b_{mi} \frac{\sigma^i \Delta u}{\delta^2} + \frac{\partial^2 f_m}{2\partial l^2} \quad (26)$$

The last term of Eq. (26), $\frac{\partial^2 f_m}{2\partial l^2}$, represents the inherent error of equilibrium manifold expansion model. Suppose that, the matrix T represents the parameter deviation, scheduling variables and their distance to equilibrium manifold; the vector K represents the required dynamic parameters, the matrix Y represents the combination of dynamic parameters and distance of scheduling variables to equilibrium manifold. So, the

identification equation can be described by

$$TK = Y + \varepsilon \quad (27)$$

The error of Eq. (27), ε , represents the expansion quadratic directional differential. If ε is a random variable, Eq. (27) can be described as a least-square problem. So the criterion function of the least-square problem is

$$J(K) = (TK - Y)^T (TK - Y) \quad (28)$$

When minimizing $J(K)$, the evaluated value of K can be derived. Suppose K_{WLS} is satisfied with

$$J(K)|_{K_{WLS}} = \min \quad (29)$$

So,

$$\left. \frac{\partial J}{\partial K} \right|_{K_{WLS}} = \frac{\partial (TK - Y)}{\partial K} \quad (30)$$

And it can get

$$(T^T T) K_{WLS} = T^T Y \quad (31)$$

Equation (31) is called canonical equation. When $T^T T$ is nonsingular, the dynamic parameters can be identified by the least-square method.

$$K = (T^T T)^{-1} T^T Y \quad (32)$$

This is the second step of identifying the equilibrium manifold expansion model. Figure 7 shows the frame of expansion model based on equilibrium manifold.

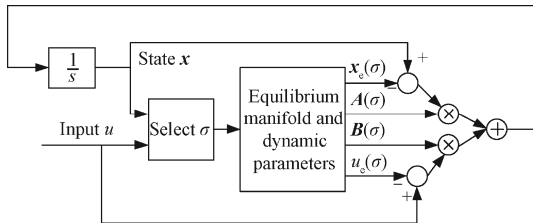


Fig. 7 Frame of expansion model based on equilibrium manifold.

4. Surge Margin Modeling and Simulation Results

4.1. Surge margin model based on equilibrium manifold expansion model

In this section, the equilibrium manifold expansion model of an aircraft engine surge margin is given. A dynamic model of a two-spool turbofan engine was developed using the MATLAB simulation environment and its Simulink toolbox. The HP compressor and HP turbine are on one shaft (driven by the high speed ro-

tor), while the low pressure (LP) compressor/fan and LP turbine are on the other shaft (driven by the low speed rotor). Components including the fan, the compressor and the turbines are described in forms of maps and look-up tables based on their individual experimental data. The combustion efficiency and pressure losses are simply fitted by curves. The single input is the fuel flow w_f . Although energy storage in and between the components are also integrated in the nonlinear thermodynamic components model, here only HP turbine speed n_H and LP turbine speed n_L are considered as states during the control oriented nonlinear modeling for their predominance. The flight condition is set to be flight attitude $H=0$ km and Mach number $Ma=0$.

The nonlinear model of HP compressor surge margin is

$$\begin{cases} \dot{n}_H = f_1(n_H, n_L, w_f) \\ \dot{n}_L = f_2(n_H, n_L, w_f) \\ SM_C = g(n_H, n_L, w_f) \end{cases} \quad (33)$$

where SM_C represents the HP compressor surge margin. The equilibrium manifold of this engine is

$$\begin{cases} f_1(n_{He}, n_{Le}, w_{fe}) = 0 \\ f_2(n_{He}, n_{Le}, w_{fe}) = 0 \\ g(n_{He}, n_{Le}, w_{fe}) = SM_{Ce} \end{cases} \quad (34)$$

The engine's Jacobian linearization form of the equilibrium manifold is shown in Eq. (35). Note that, in this paper the scheduling parameter is chosen as $\sigma = n_{He}$. With the steady state data of this aircraft engine, the other system parameters, including n_{Le} , w_f and SM_{Ce} , can be parameterized by n_{He} . The fitted curves are quadratic form shown in Eq. (36). And the curves are displayed in Figs. 8-11. According to the relationship of equilibrium manifold, we can get the perpendicular expansion shown in Eq. (37).

$$\left\{ \begin{aligned} \begin{bmatrix} \dot{n}_H \\ \dot{n}_L \end{bmatrix} &= \begin{bmatrix} \frac{\partial f_1}{\partial n_H} & \frac{\partial f_1}{\partial n_L} \\ \frac{\partial f_2}{\partial n_H} & \frac{\partial f_2}{\partial n_L} \end{bmatrix} \begin{bmatrix} n_H - n_{He}(\sigma) \\ n_L - n_{Le}(\sigma) \end{bmatrix} + \\ &\quad \begin{bmatrix} \frac{\partial f_1}{\partial w_f} \\ \frac{\partial f_2}{\partial w_f} \end{bmatrix} (w_f - w_{fe}(\sigma)) \\ SM_C &= \begin{bmatrix} \frac{\partial g}{\partial n_H} & \frac{\partial g}{\partial n_L} \end{bmatrix} \begin{bmatrix} n_H - n_{He}(\sigma) \\ n_L - n_{Le}(\sigma) \end{bmatrix} + \\ &\quad \frac{\partial g}{\partial w_f} (w_f - w_{fe}(\sigma)) + SM_{Ce} \end{aligned} \right. \quad (35)$$

$$\begin{cases} n_{Le}(\sigma) = k_{n_{L1}}\sigma^2 + k_{n_{L2}}\sigma + k_{n_{L3}} \\ w_{fe}(\sigma) = k_{w_{f1}}\sigma^2 + k_{w_{f2}}\sigma + k_{w_{f3}} \\ SM_{Ce} = k_{SM_{C1}}\sigma^2 + k_{SM_{C2}}\sigma + k_{SM_{C3}} \\ \sigma = n_H \end{cases} \quad (36)$$

$$\begin{aligned} (n_H - \sigma) + (2k_{n_{L1}}\sigma^2 + k_{n_{L2}})(n_L - k_{n_{L1}}\sigma^2 - \\ k_{n_{L2}}\sigma - k_{n_{L3}}) + (2k_{w_{f1}}\sigma^2 + k_{w_{f2}})(w_f - \\ k_{w_{f1}}\sigma^2 - k_{w_{f2}}\sigma - k_{w_{f3}}) = 0 \end{aligned} \quad (37)$$

Using the steady and dynamic data of the aircraft engine Simulink model, we can identify the coefficients of Eq. (36), and according to Eq. (37), we can get the mapping of $\sigma = p(n_H, n_L, w_f)$. The next step is to identify the Jacobian matrix. The method is the least squares identification. The form is described by Eq. (35). And the Jacobian matrix is displayed by Fig. 12.

Finally, we put $\sigma = p(n_H, n_L, w_f)$ to Eq. (35), and we can get the perpendicular expansion form of this aircraft engine.

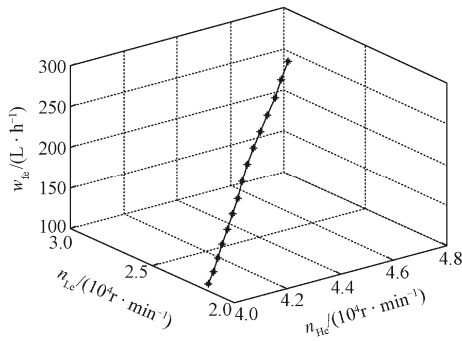


Fig. 8 Equilibrium manifold of two-spool aircraft engine.

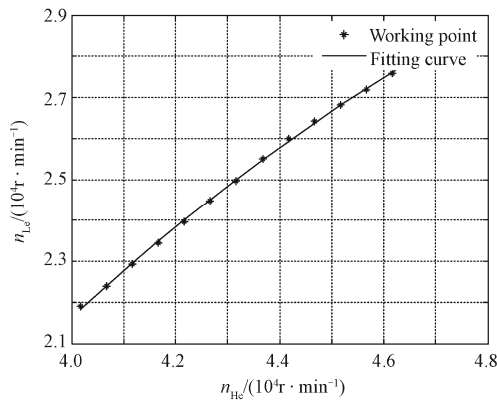


Fig. 9 Fitting result of equilibrium manifold for LP spool speed.

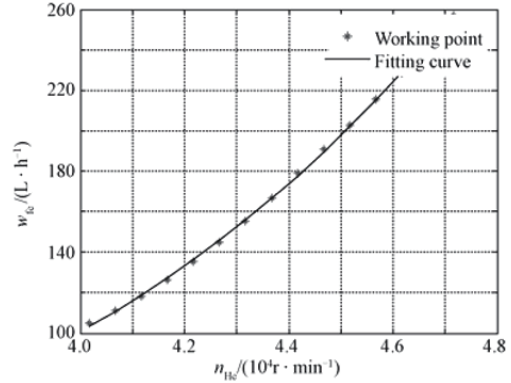


Fig. 10 Fitting result of equilibrium manifold for fuel flow.

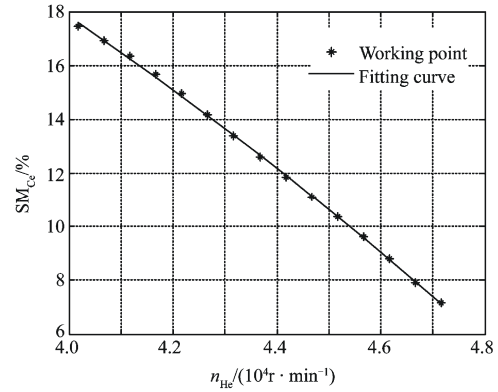


Fig. 11 Fitting result of equilibrium manifold for compressor surge margin.

4.2. Model precision

In this section, we compared the surge margin between the nonlinear thermodynamic model with the equilibrium manifold expansion model. The fuel flow command signal during two steps shown in Fig. 13 increases from 104 L/h to 250 L/h and decreases to 104 L/h.

The simulation results are shown in Figs. 14-16. With the increase in w_f , n_H is increased. The aircraft engine is working closer to the surge boundary, and the surge margin is decreased. On the contrary, with the decrease in w_f , n_H is decreased, and the surge margin is increased. When n_H 's ranges from 40 000 r/min to 46 000 r/min, the model output precision is $\pm 1.5\%$. Throughout the comparison of time consuming, the equilibrium manifold expansion model has a better real-time performance than the traditional thermodynamic model, as shown in Table 1.

As Fig. 15 shows, when the HP spool speed ranges from 40 000 r/min to 47 000 r/min, the error of model is within the range of $\pm 2\%$. The simulation result indicated that the model's accuracy can reach the engineering application requirements.

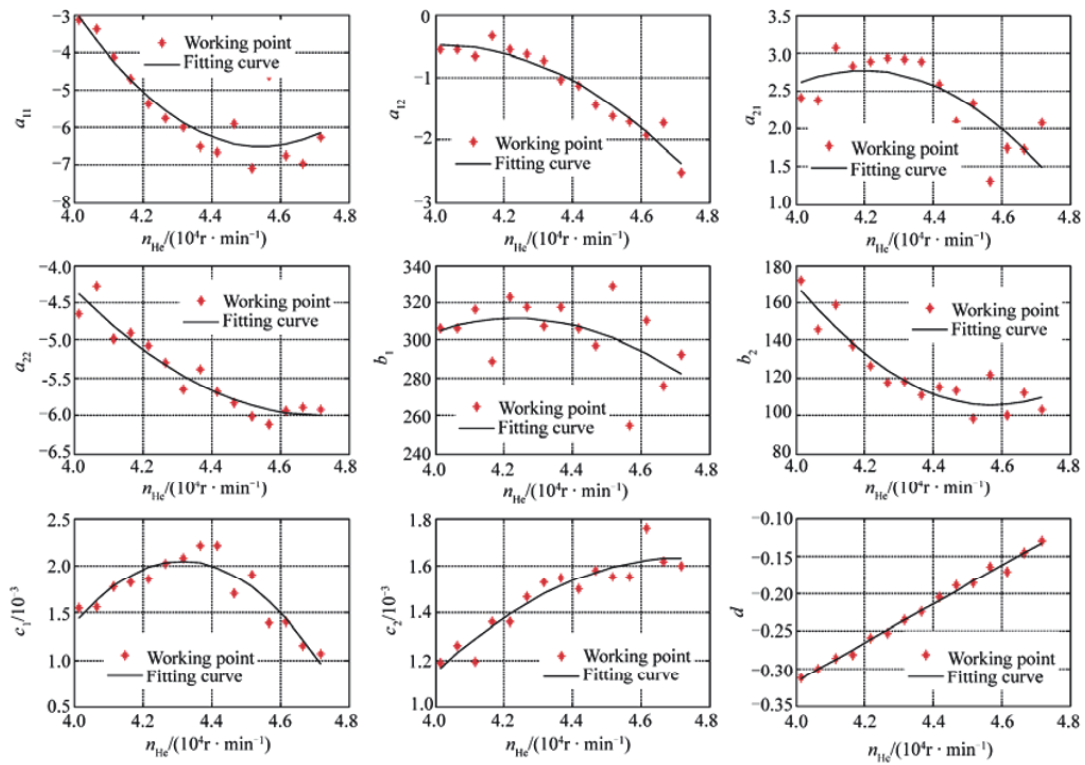


Fig. 12 Jacobian matrix of aircraft engine.

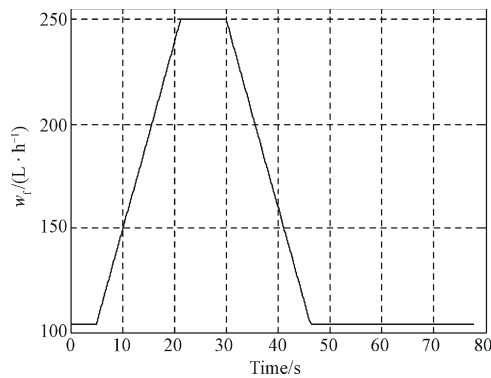


Fig. 13 Command signal of fuel flow during two steps.

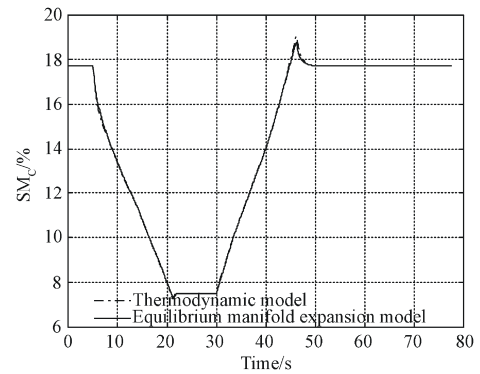


Fig. 14 Output comparison of surge margin during two steps of fuel flow.

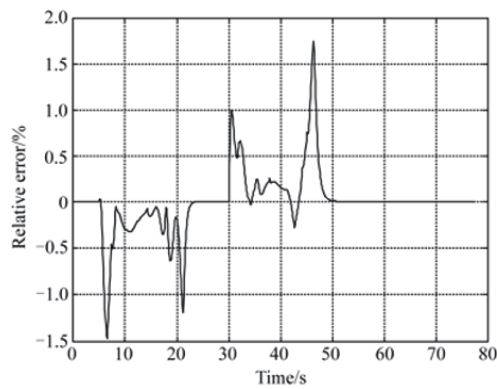


Fig. 15 Relative error.

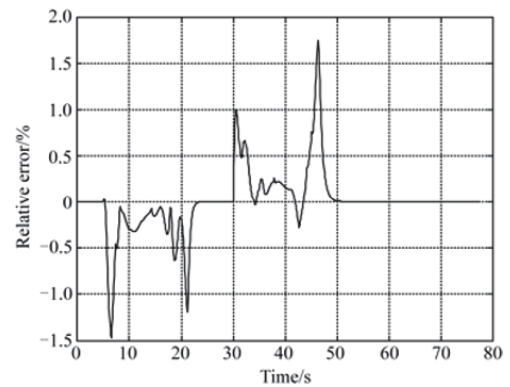


Fig. 16 Changes in HP spool speed during two steps of fuel flow.

Table 1 Comparison of simulation time

CPU		Pentium®4
Simulation condition	Main frequency/GHz	1.60
	Memory/MB	512
	Simulation time/s	60
Time consuming/s	Thermodynamic model	254
	Equilibrium manifold expansion model	10

4.3. Dynamic analysis

In this section we give two different fuel flow command signals which are shown in Fig. 17, and the dynamic process of the surge margin can be compared.

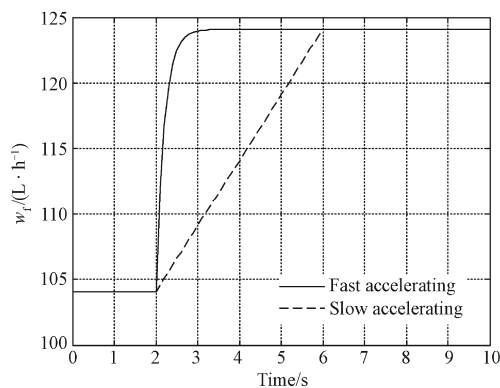


Fig. 17 Command signal of fuel flow (local enlarged drawing).

Figures 18-19 show that when given the same fuel flow command, the effect of the variation velocity is large. The fast accelerating process has a bigger overshoot (It could reach 50), and it does harm to the engine safety. When using the slow acceleration, the dynamic process of the surge margin is gentle and does less harm to the engine safety.

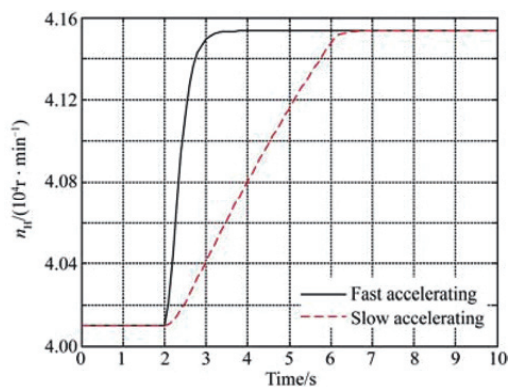


Fig. 18 Changes in HP spool speed (local enlarged drawing).

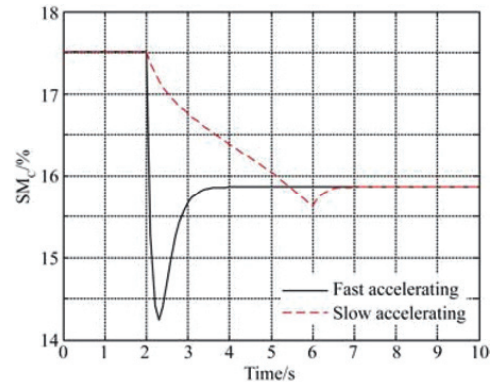


Fig. 19 Comparison of surge margin in different accelerate process.

The next simulation shows the variation of the surge margin during adding the fuel flow $\Delta w_f = 15$ L/h at different working points.

Figures 20-22 show the different surge margins during a series of fuel flow. Figures 23-24 show the local magnify figures at the first and the last working points. At the first working point, the steady variation of surge margin is 1.219 3, the dynamic time 5 s, and at the last working point, the variation is 0.943 6, the time 3.5 s. Throughout the analysis of the whole working points, when given the flight condition, the magnifying coefficient and time coefficient decrease and the dynamic response becomes faster with the increase of the spool speed.

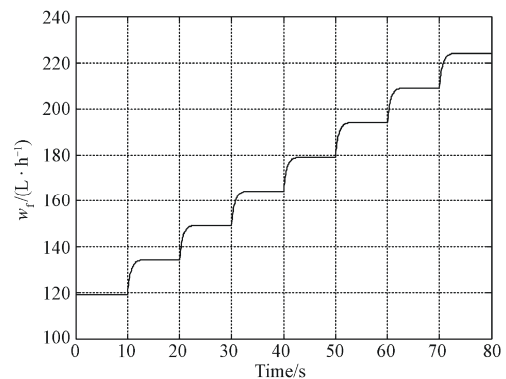


Fig. 20 Command signal of fuel flow during a series steps.

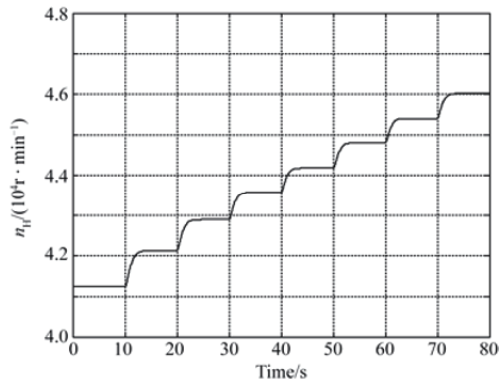


Fig. 21 Changes in HP spool speed during a series steps of fuel flow.

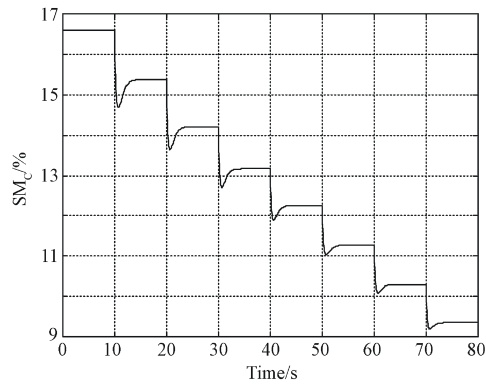


Fig. 22 Comparison of surge margin under fuel flow input.

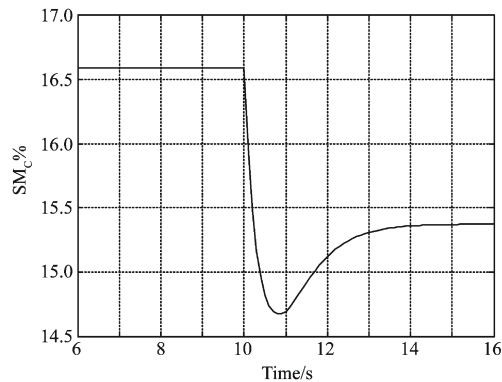


Fig. 23 Dynamic process of the first working condition (local magnify).

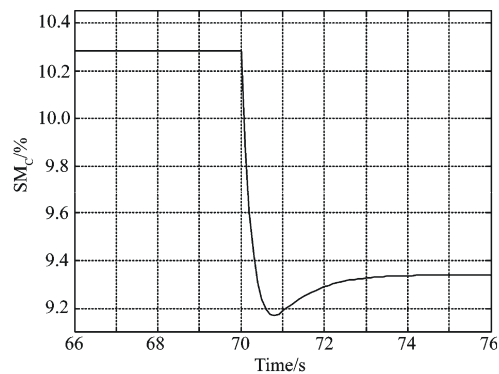


Fig. 24 Dynamic process of the last working condition (local magnify).

Figures 25-27 show the changes in HP spool speed and surge margin under different flight conditions. The simulation results indicate that the equilibrium manifold expansion model has the same accuracy with the thermodynamic model and it also shows the feasible of the method of this paper.

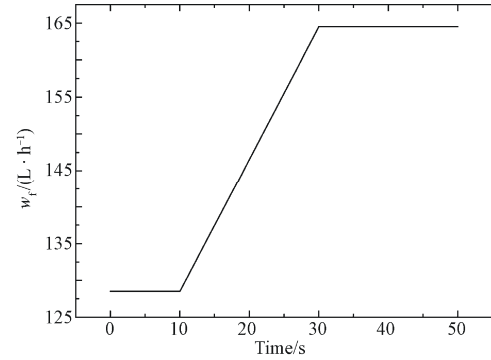


Fig. 25 Command signal of fuel flow under different flight conditions.

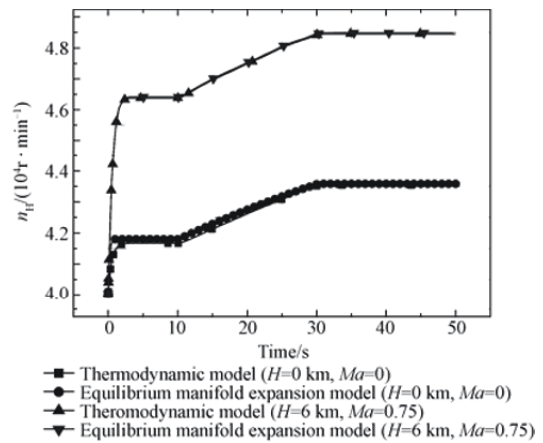


Fig. 26 Changes in HP spool speed under different flight conditions.

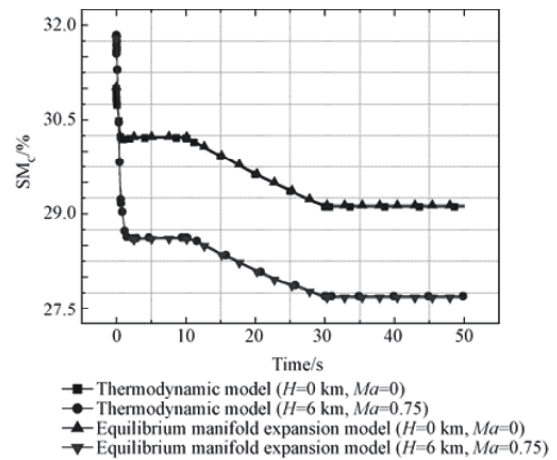


Fig. 27 Changes in surge margin of compressor under different flight conditions.

5. Conclusions

1) The equilibrium manifold expansion model introduced is an approximate nonlinear model that develops from the linearization family. The scheduling variable method is discussed, and perpendicular expansion model for nonlinear system that has the minimum error is provided. Comparisons indicate that all of the conveniently chosen mappings have given a very good modeling result.

2) Throughout the modeling of the aircraft engine surge margin based on equilibrium manifold perpendicular expansion model can be efficiently obtained by a two-step identification procedure.

3) Simulations show that the surge margin can not only be in compliance with the aircraft physical characteristics, but also satisfy the accuracy of modeling requirement. The equilibrium manifold expansion model is simple in structure and it is a real-time model.

References

- [1] Cohen H, Rogers G F C, Saravanamuttoo H I H. Gas turbine theory. 3rd ed. New York: John Wiley and Sons Inc., 1987.
- [2] Greitzer E M. Surge and rotating stall in axial flow compressors-Part I: theoretical compression system model. *ASME Journal of Engineering for Gas Turbines and Power* 1976; 98(2): 190-198.
- [3] Moore F K, Greitzer E M. A theory of post-stall transients in axial compression systems-Part I: development of equations. *ASME Journal of Engineering for Gas Turbines and Power* 1986; 108(1): 68-76.
- [4] Greitzer E M, Moore F K. A theory of post-stall transients in axial compression systems-Part II: application. *ASME Journal of Engineering for Gas Turbines and Power* 1986; 108(1): 231-239.
- [5] Feulner M R, Hendricks G J, Paduano J D. Modeling for control of rotating stall in high-speed multistage axial compressors. *ASME Journal of Turbomachinery* 1996; 118(1): 1-10.
- [6] Willems F, de Jager B. Modelling and control of rotating stall and surge: an overview. *Proceedings of the IEEE International Conference on Control Applications*, 1998; 331-335.
- [7] Gravdahl J T, Egeland O. A Moore-Greitzer axial compressor model with spool dynamics. *Proceedings of the IEEE Conference on Decision and Control*, 1997; 4714-4719.
- [8] Gravdahl J T, Egeland O. Speed and surge control for a low order centrifugal compressor model. *Proceedings of the IEEE International Conference on Control Applications*, 1997; 344-349.
- [9] Gu G, Sparks A, Banda S S. An overview of rotating stall and surge control for axial flow compressors. *IEEE Transactions on Control Systems Technology* 1999; 7(6): 639-647.
- [10] Leonessa A, Chellaboina V, Haddad W M. Multi-mode control of axial compressors via stability-based switching controllers. *Journal of Propulsion and Power* 1999; 15(2): 364-367.
- [11] van de Wal M, Willems F, de Jager B. Selection of actuators and sensors for surge control. *Journal of Propulsion and Power* 2002; 18(1): 84-92.
- [12] Zaïet C, Akhrif O, Saydy L. Modeling and nonlinear control of a gas turbine. *IEEE International Symposium on Industrial Electronics*, 2006; 4: 2688-2694.
- [13] Buhr C A, Franchek M A, Fleeter S. Rotating stall control in axial compressor subject to wheel speed transients. *Journal of Propulsion and Power* 2006; 22(2): 404-410.
- [14] Mansoux C A, Gysling D L, Setiawan J D, et al. Distributed nonlinear modeling and stability analysis of axial compressor stall and surge. *Proceedings of the American Control Conference*, 1994; 2305-2316.
- [15] Wang H H, Krstic M, Larsen M. Control of deep-hysteresis aeroengine compressors-Part I: a Moore-Greitzer type model. *Proceedings of the American Control Conference*, 1997; 2: 998-1002.
- [16] Mobarak A, Elaraby M, Shash Y. Turbo-compressors surge and surge control. *AIAA Aerospace Sciences Meeting*, 1981.
- [17] Badmus O O, Chowdhury S, Eveker K M, et al. Simplified approach for control of rotating stall-Part I: theoretical development. *Proceedings of the Joint Propulsion Conference*, 1993.
- [18] Badmus O O, Chowdhury S, Eveker K M, et al. Control-oriented high-frequency turbomachinery modeling: single-stage compression system one-dimensional model. *Journal of Turbomachinery* 1995; 117(1): 47-61.
- [19] Badmus O O, Chowdhury S, Eveker K M, et al. Simplified approach for control of rotating stall-Part II: experimental results. *Proceedings of the Joint Propulsion Conference*, 1993.
- [20] de Jager B. Rotating stall and surge control: a survey. *Proceedings of the IEEE Conference on Decision and Control*, 1995; 2: 1857-1862.
- [21] Behnken R L, D'Andrea R D, Murray R M. Control of rotating stall in a low-speed axial flow compressor using pulsed air injection: modeling, simulations, and experimental validation. *Proceedings of the IEEE Conference on Decision and Control*, 1995; 3: 3056-3061.
- [22] Sodemann A, Li Y, Lee J, et al. Data-driven surge map modeling for centrifugal air compressors. *Proceedings of International Mechanical Engineering Congress and Exposition*, 2006; 611-618.
- [23] Wu X, Li Y Y. Computationally efficient data-driven surge map modeling for centrifugal air compressors. *Proceedings of the American Control Conference*, 2007; 810-815.
- [24] Stein A, Niazi S, Sankar L N. Computational analysis of stall and separation control in centrifugal compressor. *AIAA-1998-3296*, 1998.
- [25] Yoon S Y, Lin Z L, Lim K T, et al. Model validation for an AMB-based compressor surge control test rig. *Joint 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference*, 2009; 756-761.
- [26] Stetson H D. Designing for stability in advanced turbine engines. *International Journal of Turbo and Jet Engines* 1984; 1: 235-245.
- [27] Krantz S G, Parks H R. The implicit function theorem:

history, theory, and applications. Boston: Birkhauser, 2002.

- [28] Stilwell D J, Rugh W J. Interpolation of observer state feedback controllers for gain scheduling. IEEE Transactions on Automatic Control 1999; 44(6): 1225-1229.

Biographies:

LIU Xiaofeng is a teacher and postgraduate supervisor at School of Transportation Science and Engineering, Beihang University. He received the B.S., M.S. and Ph.D. degrees in power machinery engineering from Harbin Institute of Technology in 2002, 2004 and 2008 respectively. His main research interests are dynamic modeling and control of aircraft engine.

E-mail: liuxf@buaa.edu.cn

ZHAO Lei is a postgraduate student at School of Transportation Science and Engineering, Beihang University. He received his B.S. degree from China Agricultural University in 2011. His area of research includes dynamic modeling and control of aircraft engine.

E-mail: zl5130108@126.com